

# Translation Validation using Path Based Equivalence Checkers Augmented with SMT Solvers

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# Outline

- 1 **Background**
  - Translation validation
  - Path based equivalence checkers
  - SMT solvers
- 2 **Normalization technique**
- 3 **Deploying SMT solvers**
- 4 **Experimental results**
- 5 **Conclusion and future works**



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# Background

**Program:** An organized list of instructions that, when executed, causes the computer to behave in a predetermined manner.

(source: webopedia.com)

We are not always happy with the programs we write.

**Objectives of program optimization:**

- To speed-up the computation
- To use less resource, eg. memory, power, etc.

So, we need a **compiler**.



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# Can you trust your compiler?

## Erroneous loop reversal

```
sum = 0;
for (i=0; i<N; i++) {
    sum = sum + a[i];
}
```

```
sum = 0;
for (i=N; i>=0; i--) {
    sum = sum + a[i];
} /* a[N] gets accessed */
```

**Program:** An organized list of instructions that, when executed, causes the computer to behave in a **predetermined manner**.

*A faulty compiler can alter the meaning of a program.*



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# What is the remedy?

- Verified Compiler – All optimized programs will be *correct by construction*.

Example: CompCert, INRIA

Limitations:

- Very hard to formally verify all passes of a compiler.
- Undecidability of the general problem of program verification restricts the scope of the input language supported by the verified compiler.
- Translation Validation – Each individual translation is followed by a validation phase which verifies that the target code produced correctly implements the source code.

(This is what we do, i.e., equivalence checking of programs.)



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# How to verify programs?

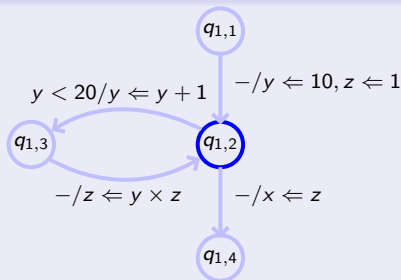
Break a program into smaller chunks — cut loops.

## Representing a program using CDFG

```

y := 10;
z := 1;
while ( y < 20 ) {
  y := y + 1;
  z := y × z;
}
x := z;

```



All computations of the program can be viewed as a concatenation of paths.

Example:  $p_1.p_3$ ,  $p_1.p_2.p_3$ ,  $p_1.p_2.p_2.p_3$ ,  $p_1.(p_2)^*.p_3$



# Finite State Machine with Datapath (FSMD)

FSMDs effectively capture both the control flow and the associated data processing of a behaviour.

The FSMD model is a seven tuple  $F = \langle Q, q_0, I, V, O, f, h \rangle$ :

$Q$ : Finite set of control states

$q_0$ : Reset state, i.e.  $q_0 \in Q$

$I$ : Set of input variables

$V$ : Set of storage variables

$O$ : Set of output variables

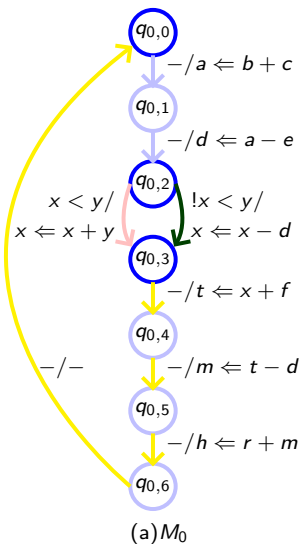
$f$ : State transition function, i.e.  $Q \times 2^S \rightarrow Q$

$h$ : Update function of the output and the storage variables, i.e.  
 $Q \times 2^S \rightarrow U$

- $U$  represents a set of storage or output assignments
- $S$  is a set of arithmetic relations between arithmetic expressions



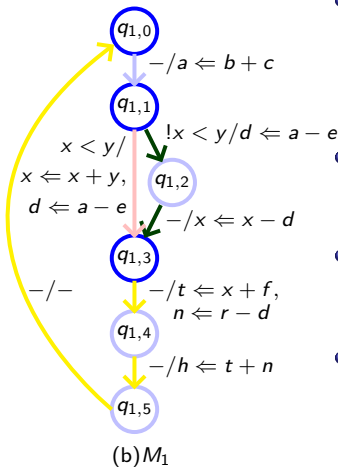
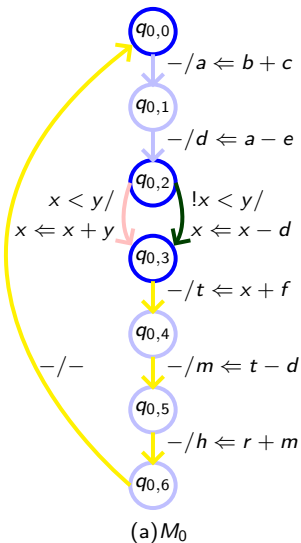
# Equivalence checking of FSMDs: A basic example



- Any computation in an FSMD can be represented by a concatenation of its computation paths
- A path is an alternating sequence of states and transitions, starting and ending at cutpoints
- Identification of suitable cutpoints and the path segments between them leads to a finite path cover  $P_0$  in  $M_0$
- For an FSMD, the reset state and all states with multiple incoming/outgoing transitions can be considered as the cutpoints
- Length and number of computations of an FSMD can both be *infinite*
- Since any computation corresponds to a concatenation of paths, it is enough to establish path equivalences



# Equivalence checking of FSMs: A basic example



- Two FSMs  $M_0$  and  $M_1$  are equivalent if for every path in  $P_0$  there is an equivalent path in  $P_1$  and vice versa

- Code transformations can make this job difficult

- Paths may be extended, and the path covers are updated accordingly

- $\{q_{0,0} \xrightarrow{x < y} q_{0,3} \simeq q_{1,0} \xrightarrow{x < y} q_{1,3}, q_{0,0} \xrightarrow{!x < y} q_{0,3} \simeq q_{1,0} \xrightarrow{!x < y} q_{1,3}, q_{0,3} \implies q_{1,3}\}$



# SMT solvers

## SMT: Satisfiability Modulo Theories

The SMT problem is a decision problem for logical formulas with respect to combinations of background theories expressed in classical first-order logic with equality.

(source: wikipedia.org)

Example:  $3x + 2y \geq 4$ ,  $x, y \in \mathbb{N}$

SMT solvers used in this work: CVC4, Yices2, Z3

Other SMT solvers: Beaver, Boolector, MiniSmt, SONOLAR





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# How to establish equivalence of expressions?

$$X + \bar{Y}.Z \equiv X + X.\bar{Y}.Z + \bar{X}.\bar{Y}.Z ?$$

Convert both expressions into sum-of-minterms

$$X.(Y + \bar{Y}).(Z + \bar{Z}) + \bar{Y}.Z.(X + \bar{X}) \equiv \\ X.(Y + \bar{Y}).(Z + \bar{Z}) + X.\bar{Y}.Z + \bar{X}.\bar{Y}.Z ?$$

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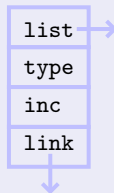


# A normalization technique for integers

No canonical representation exists for expressions over integers.

## Structure of a normalized cell

```
typedef struct normalized_cell NC;  
  
struct normalized_cell {  
    NC *list;  
    char type;  
    int inc;  
    NC *link;  
};
```



Proposed by *J. C. King*, "A Program Verifier," PhD thesis, Carnegie-Mellon University, 1969.

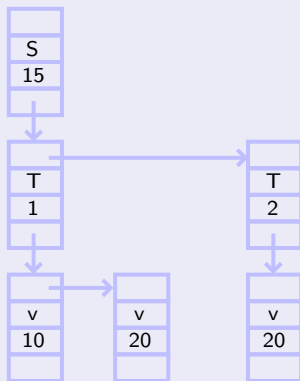


# An example of normalized expression over integers

Expression:  $1 \times y \times z + 2 \times z + 15$

Symbols

Loc	Var
y	10
z	20



Normalization can show that this expression is equivalent to  $z + z \times y + z + 20 - 5$ .



# Normalization grammar

## Grammar

- 1)  $S \rightarrow S + T \mid c_s$ , where  $c_s$  is an integer.
- 2)  $T \rightarrow T * P \mid c_t$ , where  $c_t$  is an integer.
- 3)  $P \rightarrow \text{abs}(S) \mid (S) \bmod(S) \mid S \div C_d \mid v \mid c_p$ ,  
where  $v \in I \cup V$ , and  $c_p$  is an integer.
- 4)  $C_d \rightarrow S \div C_d \mid S$ .

Some simplification rules for integers are given in [TCAD08].  
This grammar is latter applied on reals also in [TODAES12].



# Limitations of the normalization method

## An example where normalization fails

```
if( a != b ) {
  n := a×a - 2×a×b + b×b;
  d := a - b;
  x := n / d;
}

if( a != b ) {
  x := a - b;
}
```

- The normalization technique resolves equivalence of expressions by reducing them to the same syntactical structure and does not actually *solve* the expressions by substituting for variables.
- The normalization technique does not account for bit-vectors and user-defined datatypes.



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# Single assignment form: A prerequisite for SMT Solvers

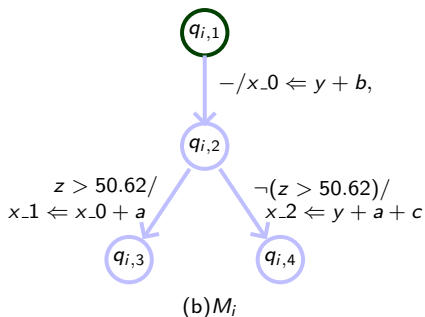
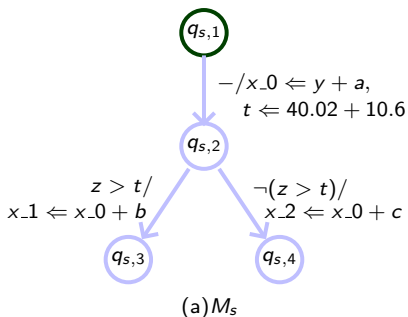
## An example to highlight single assignment form

S1: $x := a + b;$	$x_0 := a + b;$	ASSERT $x_0 = a + b;$
S2: $x := x + c;$	$x_1 := x_0 + c;$	ASSERT $x_1 = x_0 + c;$
S3: $y := x + d;$	$y := x_1 + d;$	ASSERT $y = x_1 + d;$
(a)	(b)	(c)

- The *order of execution* of the statements is not captured by the ordering of the *assert* statements.
- Programs in single assignment form help in producing *assert* statements whose ordering is irrelevant, that is, they can be arranged in any order to produce the same effect.



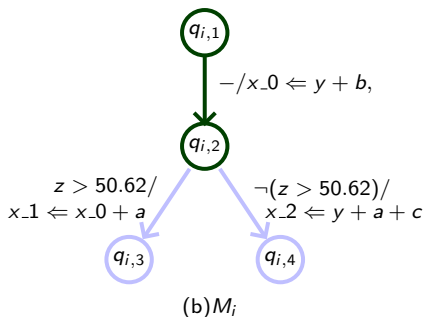
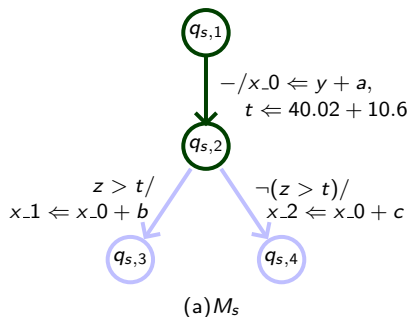
# Formula generation for SMT solvers



Here, we have considered the path based equivalence checker of [ISED12].



# Formula generation for SMT solvers



## Encoding in CVC4 input language

```

y_s:INT; a_s:INT; x_0_s:INT; t_s:REAL;
y_i:INT; b_i:INT; x_0_i:INT;
ASSERT y_s = y_i;
ASSERT x_0_s = y_s+a_s; ASSERT t_s = 40.02 + 10.6;
ASSERT x_0_i = y_i + b_i;
QUERY x_0_s = x_0_i;

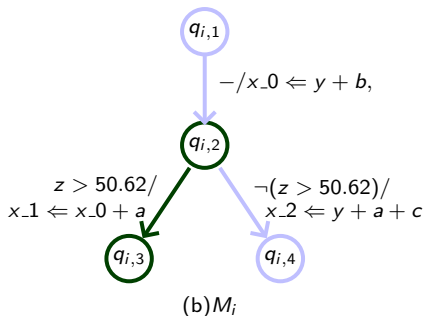
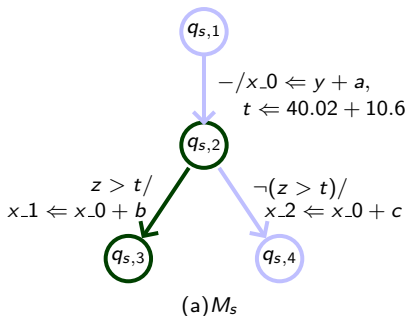
```

Output: invalid (need to look beyond this basic block)





# Formula generation for SMT solvers



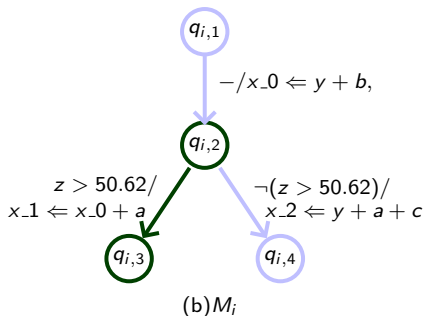
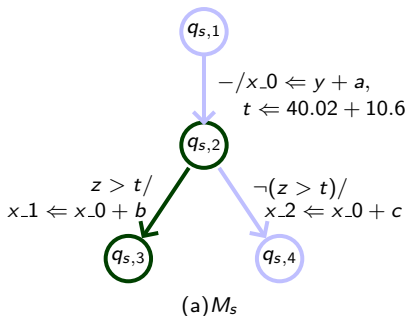
Encoding in CVC4 input language (appended with the previous one)

```
z_s:REAL; cond_s:BOOLEAN;
z_i:REAL; cond_i:BOOLEAN;
ASSERT z_s = z_i;
ASSERT cond_s = z_s > t_s;
ASSERT cond_i = z_i > 50.62;
QUERY cond_s = cond_i;
```

Output: valid (these two branches run in synchrony)



# Formula generation for SMT solvers



Encoding in CVC4 input language (appended with the earlier one)

```

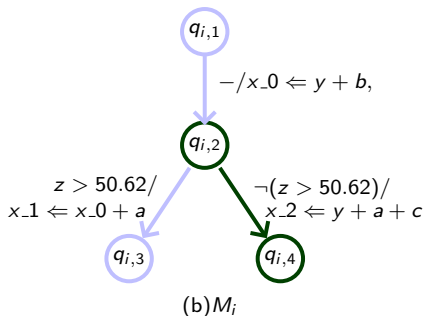
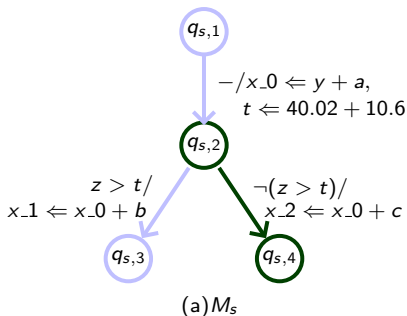
b_s:INT; x_1_s:INT;
a_i:INT; x_1_i:INT;
ASSERT a_s = a_i; ASSERT b_s = b_i;
ASSERT x_1_s = x_0_s + b_s;
ASSERT x_1_i = x_0_i + a_i;
QUERY x_1_s = x_1_i;

```

Output: valid (the computations match at states  $q_{s,3}$  and  $q_{i,3}$ )



# Formula generation for SMT solvers



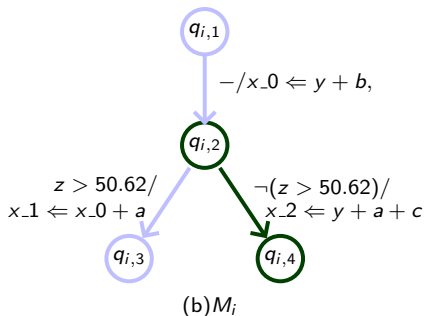
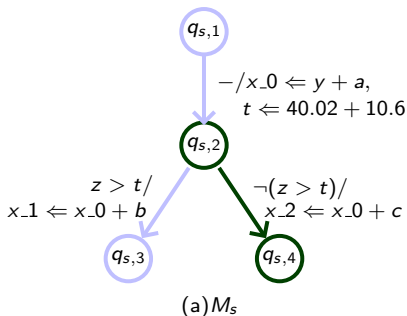
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z_s:REAL; cond_s:BOOLEAN;
z_i:REAL; cond_i:BOOLEAN;
ASSERT z_s = z_i;
ASSERT cond_s = z_s <= t_s;
ASSERT cond_i = z_i <= 50.62;
QUERY cond_s = cond_i;
```

Output: valid (these two branches run in synchrony)



# Formula generation for SMT solvers



Encoding in CVC4 input language (appended with the earliest one)

```

c_s:INT; x_2_s:INT;
a_i:INT; c_i:INT; x_2_i:INT;
ASSERT a_s = a_i; ASSERT b_s = b_i; ASSERT c_s = c_i;
ASSERT x_2_s = x_0_s + c_s;
ASSERT x_2_i = y_i + a_i + c_i;
QUERY x_2_s = x_2_i;

```

Output: valid (the computations match at states  $q_{s,4}$  and  $q_{i,4}$ )



# Revisiting the example where normalization fails

## An example where normalization fails

```
if( a != b ) {
  n := a×a - 2×a×b + b×b;
  d := a - b;
  x := n / d;
}
```

```
if( a != b ) {
  x := a - b;
}
```

## Encoding in SMT2 input language

```
(declare-const a_s Real) (declare-const b_s Real) (declare-const n_s Real)
(declare-const d_s Real) (declare-const x_s Real)
(declare-const a_i Real) (declare-const b_i Real) (declare-const x_i Real)
(assert (= a_s a_i)) (assert (= b_s b_i))
(assert (not (= a_s b_s)))
(assert (= n_s (+ (- (* a_s a_s) (* 2 a_s b_s)) (* b_s b_s))))
(assert (= d_s (- a_s b_s))) (assert (= x_s (/ n_s d_s)))
(assert (not (= a_i b_i))) (assert (= x_i (- a_i b_i)))
(assert (not (= x_s x_i)))
(check-sat)
```

Output of Z3: unsat



# Modeling bit-vectors and user-defined datatypes

## Bit-vector example for Z3: DeMorgan's law

```
(declare-const x (_ BitVec 64))
(declare-const y (_ BitVec 64))
(assert (not (= (bvand (bvnot x) (bvnot y)) (bvnot (bvor x y)))))
(check-sat)
```

## Declaring user-defined datatype in CVC4

```
struct recordType {
    _Bool flag;
    double r;
    int i;
};

recordType: TYPE = [#
    flag:BOOLEAN,
    r:REAL,
    i:INT
#];
```



# Outline

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  - Path based equivalence checkers
  - SMT solvers
- 2 Normalization technique
- 3 Deploying SMT solvers
- 4 **Experimental results**
- 5 Conclusion and future works



# Experimental results

**Table:** Results for our method on different benchmarks

Benchmarks	Benchmark Characteristics							Formulae		Execution Time (ms)			
	#op	#BB	#if	#loop	#path	#state <sub><math>\alpha</math></sub>	#state <sub><math>\beta</math></sub>	#assert	#query	Norm	Yices2	CVC4	Z3
DCT	42	1	0	0	1	43	12	92	8	32	54	52	42
DIFFEQ	20	3	0	1	3	18	11	67	10	13	NLA	38	39
EWF	52	1	0	0	1	30	19	113	8	63	NLA	285	161
PERFECT	12	6	3	1	7	12	10	50	14	8	NLA	22	40
PRIMEFAC	10	4	2	1	5	8	7	40	10	7	NLA	16	24
BV-DEMORGAN	9	4	1	0	3	7	6	49	15	×	13	24	34
BV-BOOLRULE	9	4	2	0	5	7	7	43	11	×	36	19	26
UD-SIMPLIFY	15	1	0	0	1	8	4	29	4	×	NLA	9	6
UD-MINMAX	15	6	3	1	7	15	11	86	22	×	32	19	33

× – Normalization technique is not applicable for these cases.

NLA – Yices2 terminated prematurely due to the presence of non-linear arithmetic.





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# Conclusion and future works

## Conclusion

- We have augmented a path based equivalence checker [ISED12] with SMT solvers.
- Experiments carried out using three SMT solvers – Yices2, CVC4, Z3 – demonstrate that the current equivalence checker is now equipped to handle bit-vectors, user-defined datatypes and sophisticated code transformations.
- The upgraded equivalence checker will automatically benefit from the current research focusing on improving (underlying) SMT solvers.
- To reduce execution time, it may be more advantageous solution to employ an SMT solver only when normalization fails to prove the equivalence.

## Future works

- Automate the whole verification process; COmpiler INfraStructure [COINS] may be helpful in this regard.
- Perform extensive experimentation to test the limits of SMT solvers.
- Since different SMT solvers excel in different fields, find out the best possible combination.



# References

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*Thank you!*

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